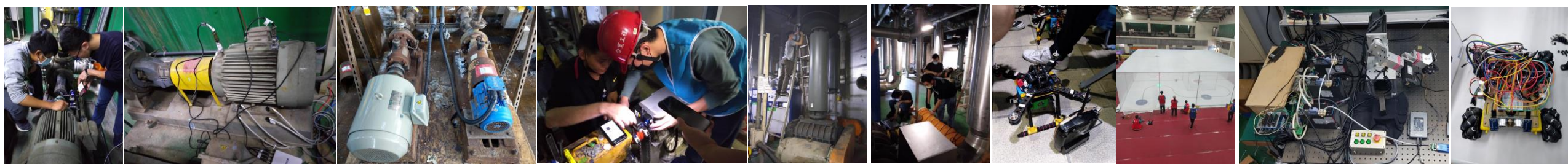
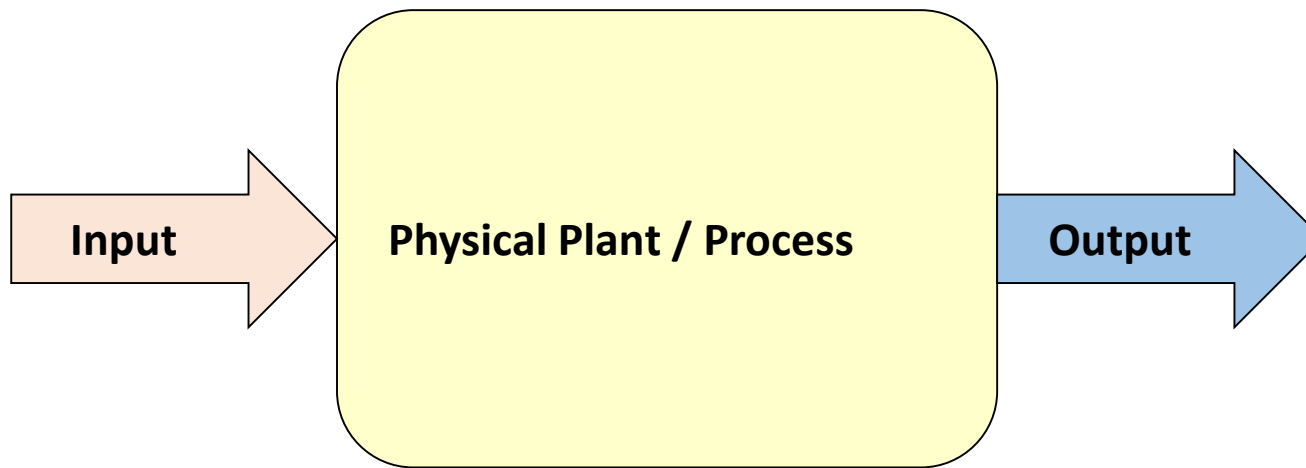


# 電機機械原理與智慧監測 Electromechanical Principles and Intelligent Monitoring

國立臺灣科技大學機械工程系

助理教授：藍振洋





---

# 最小平方法

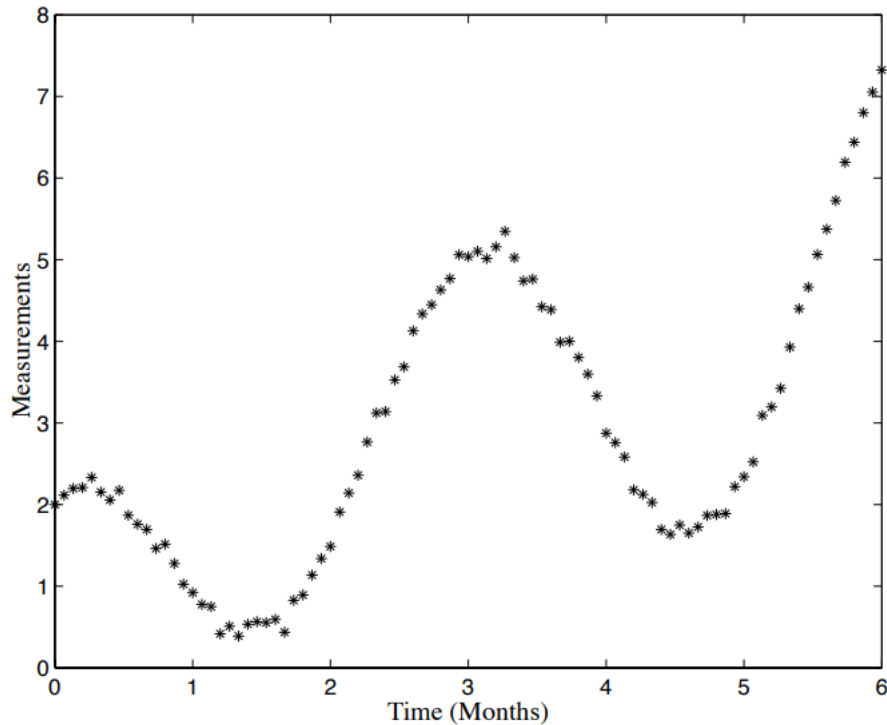
# 最小平方法

$$\begin{array}{rccccccc} \text{measured value} & = & \text{true value} & + & \text{measurement error} \\ \tilde{x} & = & x & + & v \end{array}$$

$$\begin{array}{rccccccc} \text{measured value} & = & \text{estimated value} & + & \text{residual error} \\ \tilde{x} & = & \hat{x} & + & e \end{array}$$



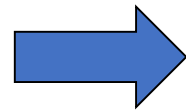
# 最小平方法



Model 1:  $y_1(t) = c_1 t + c_2 \sin(t) + c_3 \cos(2t)$

Model 2:  $y_2(t) = d_1(t+2) + d_2 t^2 + d_3 t^3$

Model 3:  $y_3(t) = x_1 t + x_2 \sin(t) + x_3 \cos(2t) + x_4 e^t$



Least Square

where  $\hat{y}(t)$  denotes the estimate of  $y(t)$

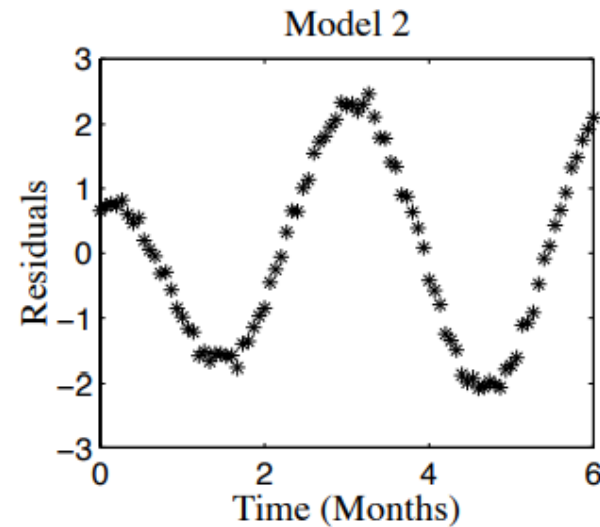
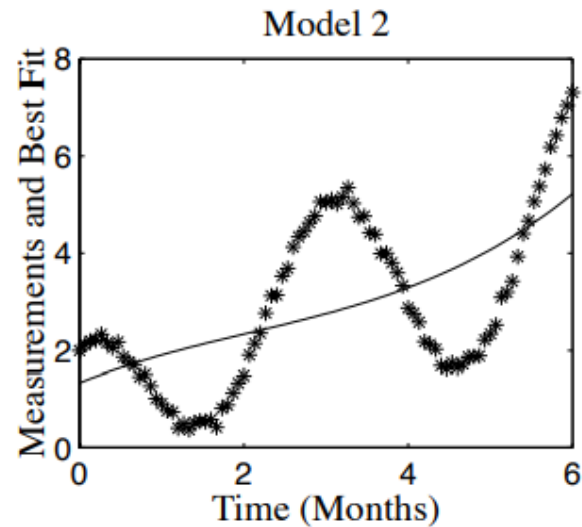
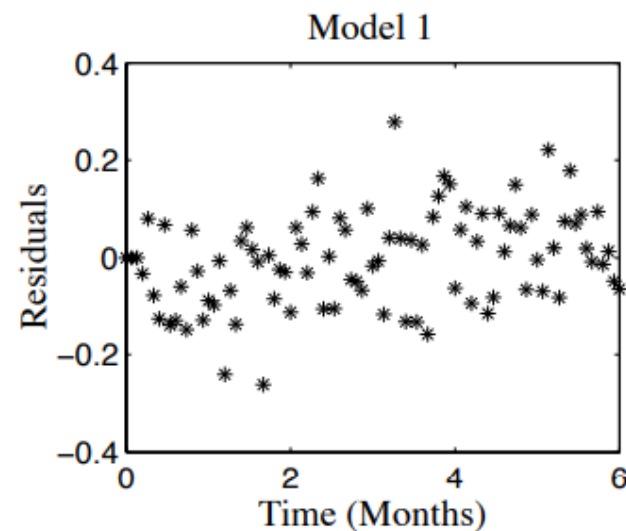
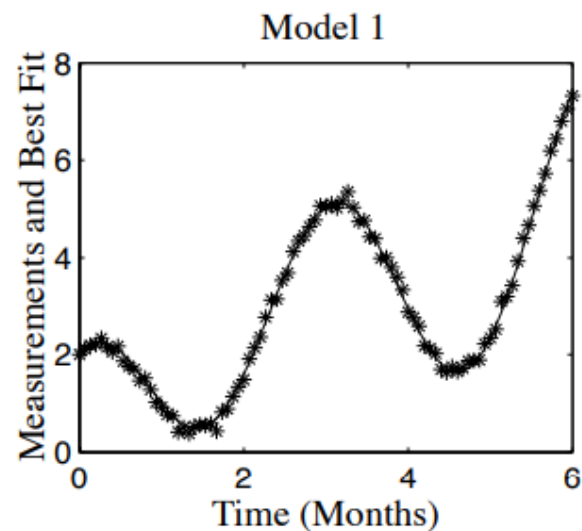
$$(\hat{c}_1, \hat{c}_2, \hat{c}_3) = (0.9967, 0.9556, 2.0030)$$

$$(\hat{d}_1, \hat{d}_2, \hat{d}_3) = (0.6721, -0.1303, 0.0210)$$

$$(\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4) = (0.9958, 0.9979, 2.0117, -4.232 \times 10^{-5})$$

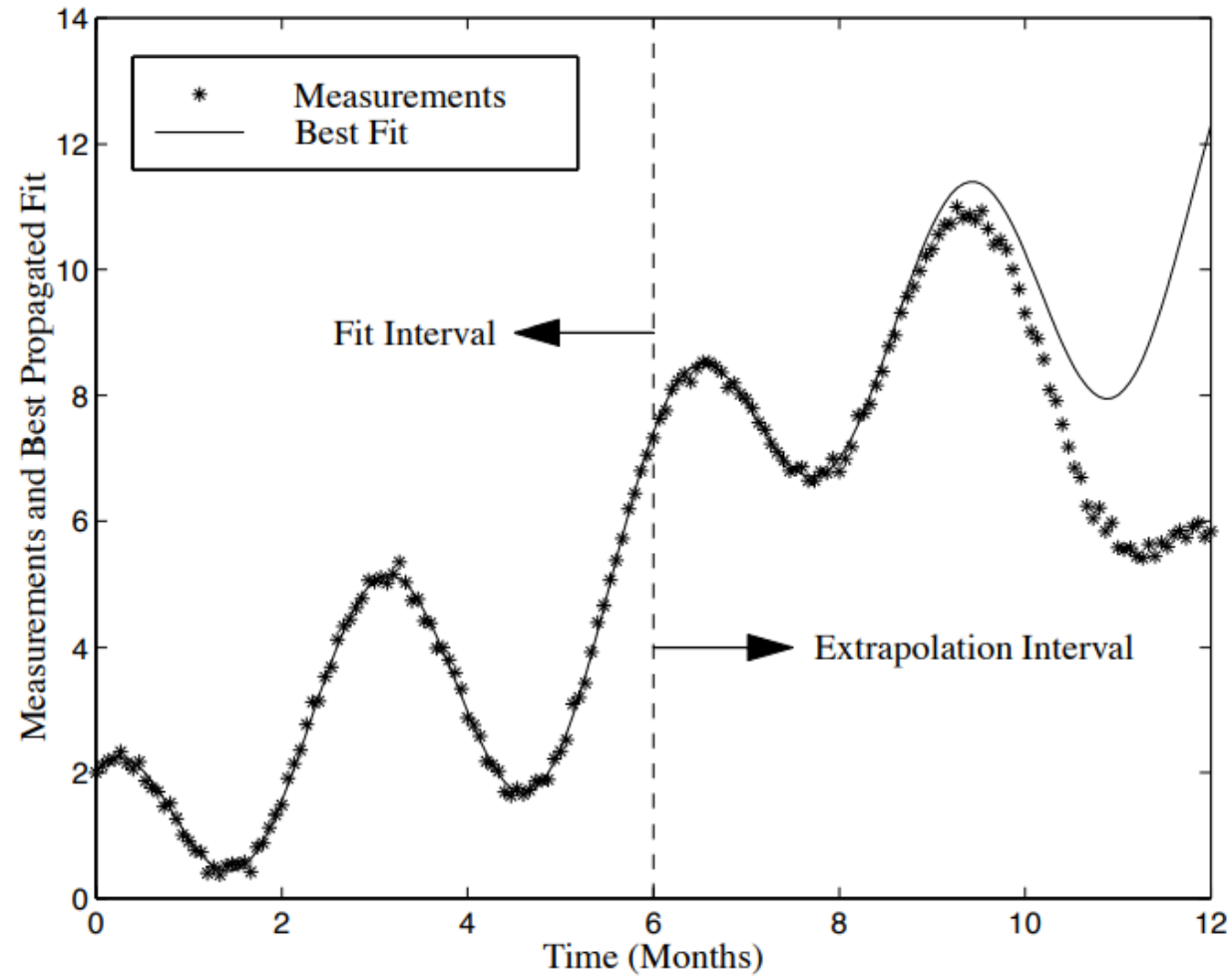
Ref : Crassidis, John L., and John L. Junkins. Optimal estimation of dynamic systems. Chapman and Hall/CRC, 2004.

# 最小二乘法



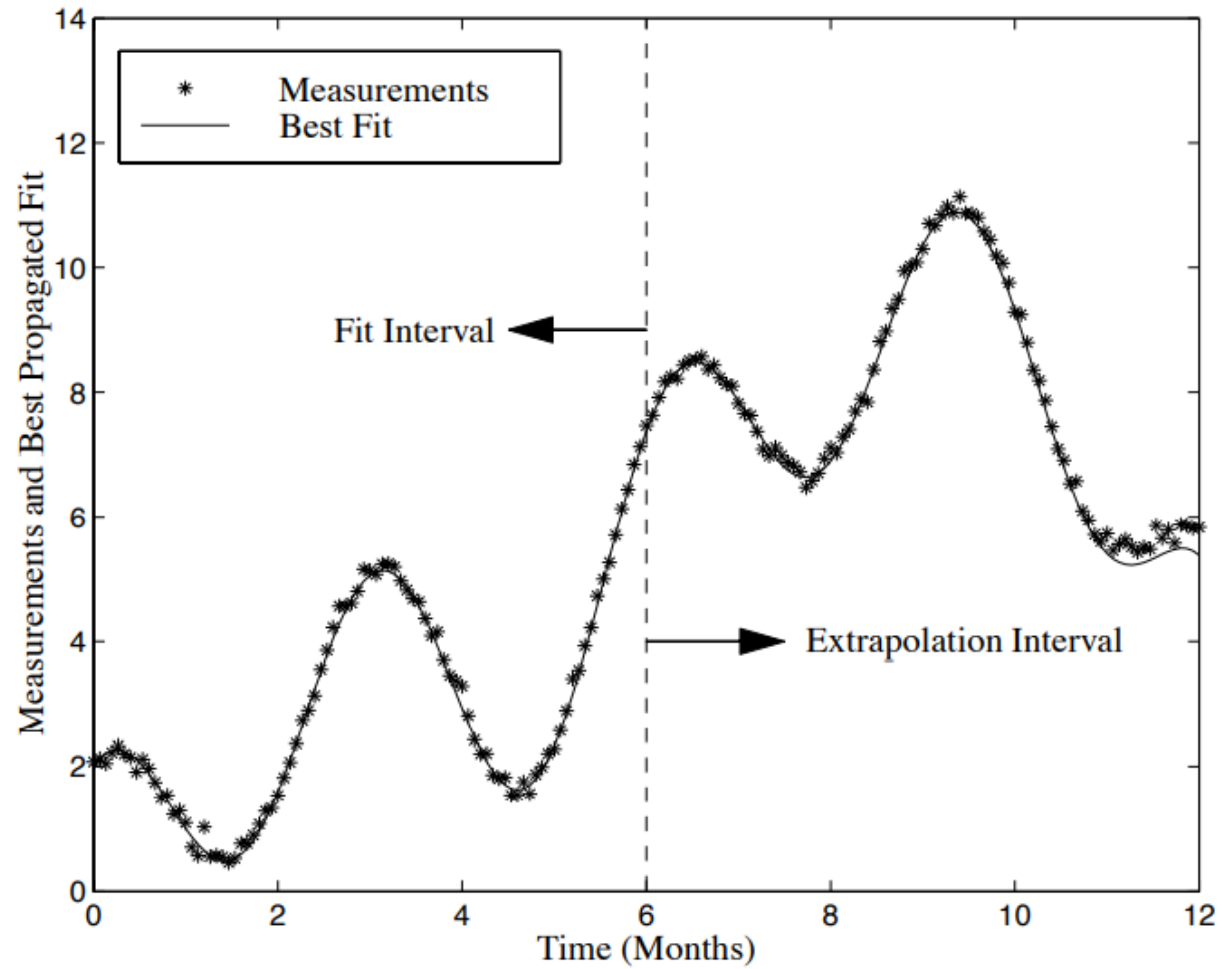
# 最小平方法

Model 1.



# 最小平方法

Model 3.





# 最小平方法

Linear Batch Estimation:

Data

$$\{\tilde{y}_1, t_1; \tilde{y}_2, t_2; \dots; \tilde{y}_m, t_m\}$$

Mathematical Model

$$y(t) = \sum_{i=1}^n x_i h_i(t), \quad m \geq n$$

$$\tilde{y}_j \equiv \tilde{y}(t_j) = \sum_{i=1}^n x_i h_i(t_j) + v_j, \quad j = 1, 2, \dots, m$$

$$\hat{y}_j \equiv \hat{y}(t_j) = \sum_{i=1}^n \hat{x}_i h_i(t_j), \quad j = 1, 2, \dots, m$$

$$\tilde{y}_j = \sum_{i=1}^n \hat{x}_i h_i(t_j) + e_j, \quad j = 1, 2, \dots, m$$

measurement error

a compact matrix form

$$\tilde{\mathbf{y}} = H\mathbf{x} + \mathbf{v}$$

$$\hat{\mathbf{y}} = H\hat{\mathbf{x}}$$

$$\tilde{\mathbf{y}} = H\hat{\mathbf{x}} + \mathbf{e}$$

residual error

# 最小平方法

$$\tilde{\mathbf{y}} = H\mathbf{x} + \mathbf{v}$$

$$\hat{\mathbf{y}} = H\hat{\mathbf{x}}$$

$$\mathbf{x} = [x_1 \ x_2 \ \cdots \ x_n]^T = \text{true } x\text{-values}$$

$$\mathbf{v} = [v_1 \ v_2 \ \cdots \ v_m]^T = \text{measurement errors}$$

$$\hat{\mathbf{y}} = [\hat{y}_1 \ \hat{y}_2 \ \cdots \ \hat{y}_m]^T = \text{estimated } y\text{-values}$$

$$\tilde{\mathbf{y}} = [\tilde{y}_1 \ \tilde{y}_2 \ \cdots \ \tilde{y}_m]^T = \text{measured } y\text{-values}$$

$$\tilde{\mathbf{y}} = H\hat{\mathbf{x}} + \mathbf{e}$$

$$\tilde{\mathbf{y}} = [\tilde{y}_1 \ \tilde{y}_2 \ \cdots \ \tilde{y}_m]^T = \text{measured } y\text{-values}$$

$$\mathbf{e} = [e_1 \ e_2 \ \cdots \ e_m]^T = \text{residual errors}$$

$$\hat{\mathbf{x}} = [\hat{x}_1 \ \hat{x}_2 \ \cdots \ \hat{x}_n]^T = \text{estimated } x\text{-values}$$

$$H = \begin{bmatrix} h_1(t_1) & h_2(t_1) & \cdots & h_n(t_1) \\ h_1(t_2) & h_2(t_2) & \cdots & h_n(t_2) \\ \vdots & \vdots & & \vdots \\ h_1(t_m) & h_2(t_m) & \cdots & h_n(t_m) \end{bmatrix}$$

# 最小平方法

an optimum choice for the unknown parameters

Cost function  $J = \frac{1}{2} \mathbf{e}^T \mathbf{e}$

$$J = J(\hat{\mathbf{x}}) = \frac{1}{2} (\tilde{\mathbf{y}}^T \tilde{\mathbf{y}} - 2\tilde{\mathbf{y}}^T H \hat{\mathbf{x}} + \hat{\mathbf{x}}^T H^T H \hat{\mathbf{x}})$$

*necessary condition*

$$\nabla_{\hat{\mathbf{x}}} J \equiv \begin{bmatrix} \frac{\partial J}{\partial \hat{x}_1} \\ \vdots \\ \frac{\partial J}{\partial \hat{x}_n} \end{bmatrix} = H^T H \hat{\mathbf{x}} - H^T \tilde{\mathbf{y}} = \mathbf{0} \quad \rightarrow \quad \boxed{\hat{\mathbf{x}} = (H^T H)^{-1} H^T \tilde{\mathbf{y}}}$$

*sufficient condition*

$$\nabla_{\hat{\mathbf{x}}}^2 J \equiv \frac{\partial^2 J}{\partial \hat{\mathbf{x}} \partial \hat{\mathbf{x}}^T} = H^T H \text{ must be positive definite}$$

is always positive semi-definite

It becomes positive definite when H is of maximum rank (n).

# 最小平方法

- 最小平方法是一種常見的參數鑑別之方法。
- 最小平方法可求線性方程式之最佳解。範例：某次實驗獲得四筆資料 $(x, y)$ ： $(1, 6)$ 、 $(2, 5)$ 、 $(3, 7)$ 、 $(4, 10)$ ，求出最匹配的直線： $y =$

$$mx + b$$

$$\begin{aligned} 6 &= 1m + b \\ 5 &= 2m + b \\ 7 &= 3m + b \\ 10 &= 4m + b \end{aligned}$$

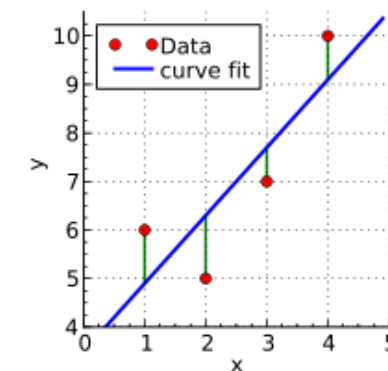
$$\rightarrow \begin{bmatrix} 6 \\ 5 \\ 7 \\ 10 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} m & b \end{bmatrix}$$

$$\mathbf{Y} = \mathbf{H} \cdot \mathbf{X}$$

量測值 = 基底函數 · 參數

$$\mathbf{X} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{Y} \quad (2)$$

$$\mathbf{X} = [1.4 \quad 3.5]$$





---

# 量測驗證基本概念

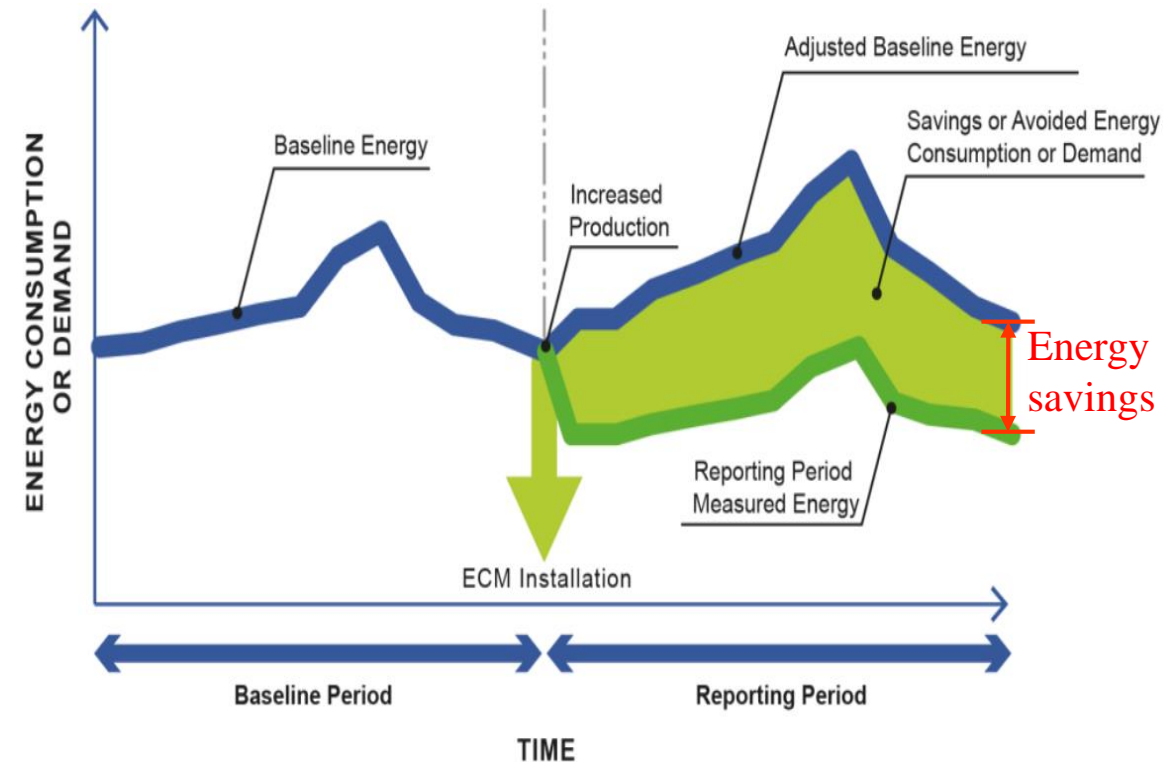
# 量測驗證基本概念

- 最近幾年，世界各國開始實施低碳且節能的政策。
- 國際能效評估組織(EVO) 的願景為建立一個全球性市集，使得對自然資源利用效率可以作正確評價，且善用提升終端效率方案成為供應端方案的重要替代選項。
- 國際能效評估組織(EVO)也致力發展並推廣標準化的協定、方法與工具，用來量化及管理伴隨能源效率改善、再生能源利用和用水效率改善等商業交易行為而來的績效風險與利益。
- 因此在2001年EVO推動IPMVP，而IPMVP可以做出公正、可驗證的節能報告，並且提供測量精度和分析，該驗證方法可以幫助到企業確認節能改進績效已達到預期目標，從而提高能源效率和減少環境影響。
- 綠基會則是在台灣推動IPMVP之量測驗證手法。



# 量測驗證基本概念

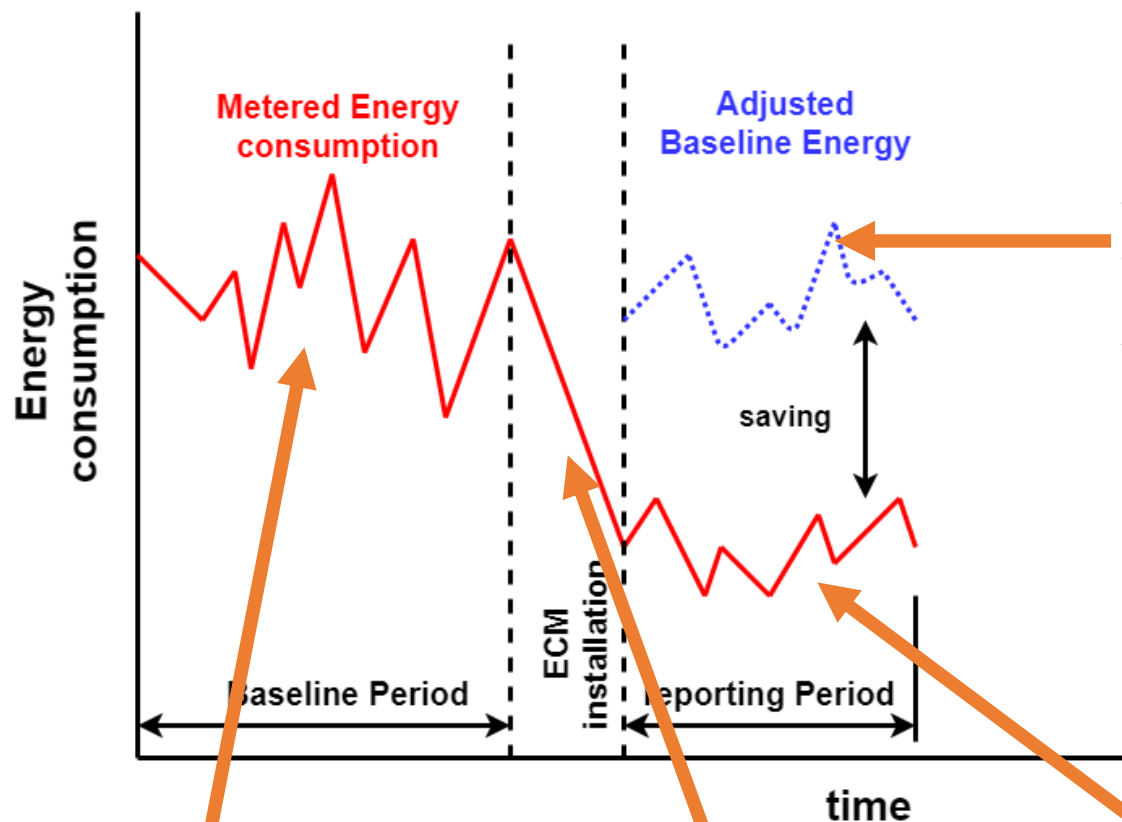
- IPMVP背景與基本概念
- 量測驗證(M&V)是一個為了驗證與報告個別設施實施節能改造措施之節能效益，所規劃、量測、收集及分析數據的一個過程。
- IPMVP之效益：
  - 定義「量測」及「節能效益」的標準程序，使業主有保障。
  - 提供量測「精度」和「成本」之間的權衡指導原則。
  - 關於節能效益的清算，幫助參與者建立節能計算透明化、可再現的能效合約條件及排放交易。
  - 提供一般性的而不是具體的指導原則，及一個倍創建且被使用，有特定方法論的框架。



$$\text{Energy Savings} = \text{Baseline Energy} - \text{Reporting Period Energy} \pm \text{Adjustments}$$



# 量測驗證基本概念



改善完的數據  
套用到改善前  
計算出的能耗  
模型所得到的  
估測數據

- 使用適當的量測週期可以確保模型的不確定度並避免不必要的成本。
- 使用過長的量測週期可能會因還進的變化導致不確定性增加，進而導致模型變得更糟。

$$\text{節能率} = \frac{(\text{舊設備推估耗能}) - \text{新設備實際耗能}}{\text{舊設備推估耗能}} \times 100\%$$

改善前我們量  
測到的數據

改善中

改善完我們  
量測到的數據

# 量測驗證基本概念

- 能耗量測驗證並非設備效率驗證
- 依據設備運轉負載狀況，評估節能改善作為所達成的節能率
- 建立舊設備用電對負載之能耗回歸模型，代入新設備之運轉負載參數，推估舊設備於新設備運轉參數下的能耗與新設備能耗比較

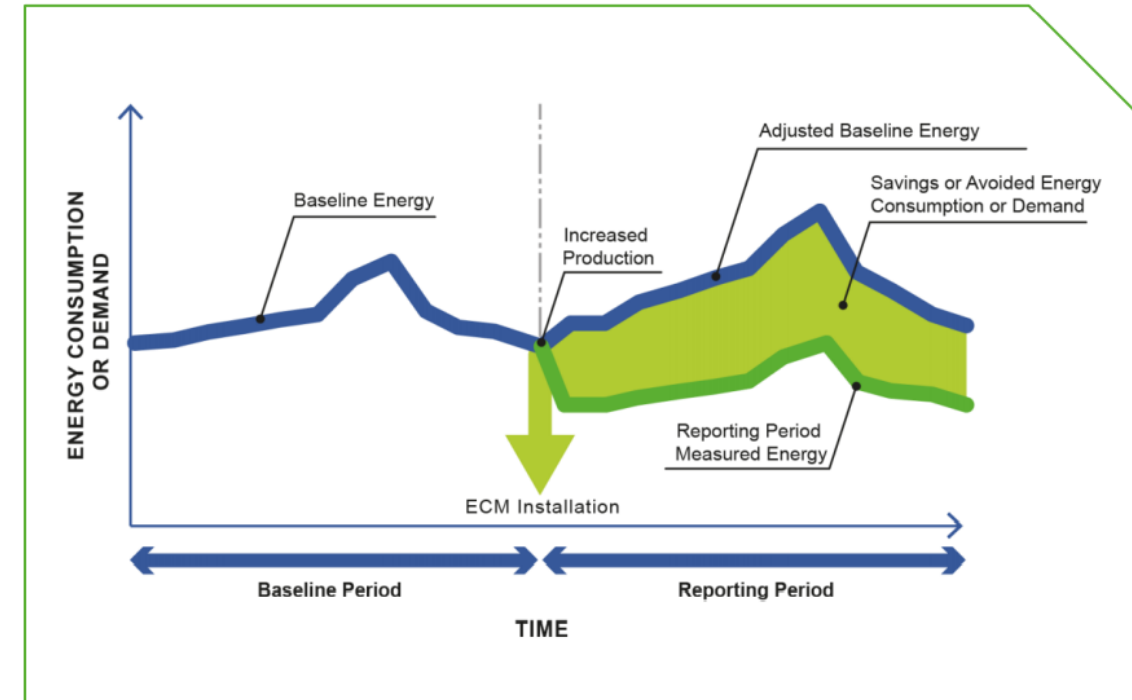


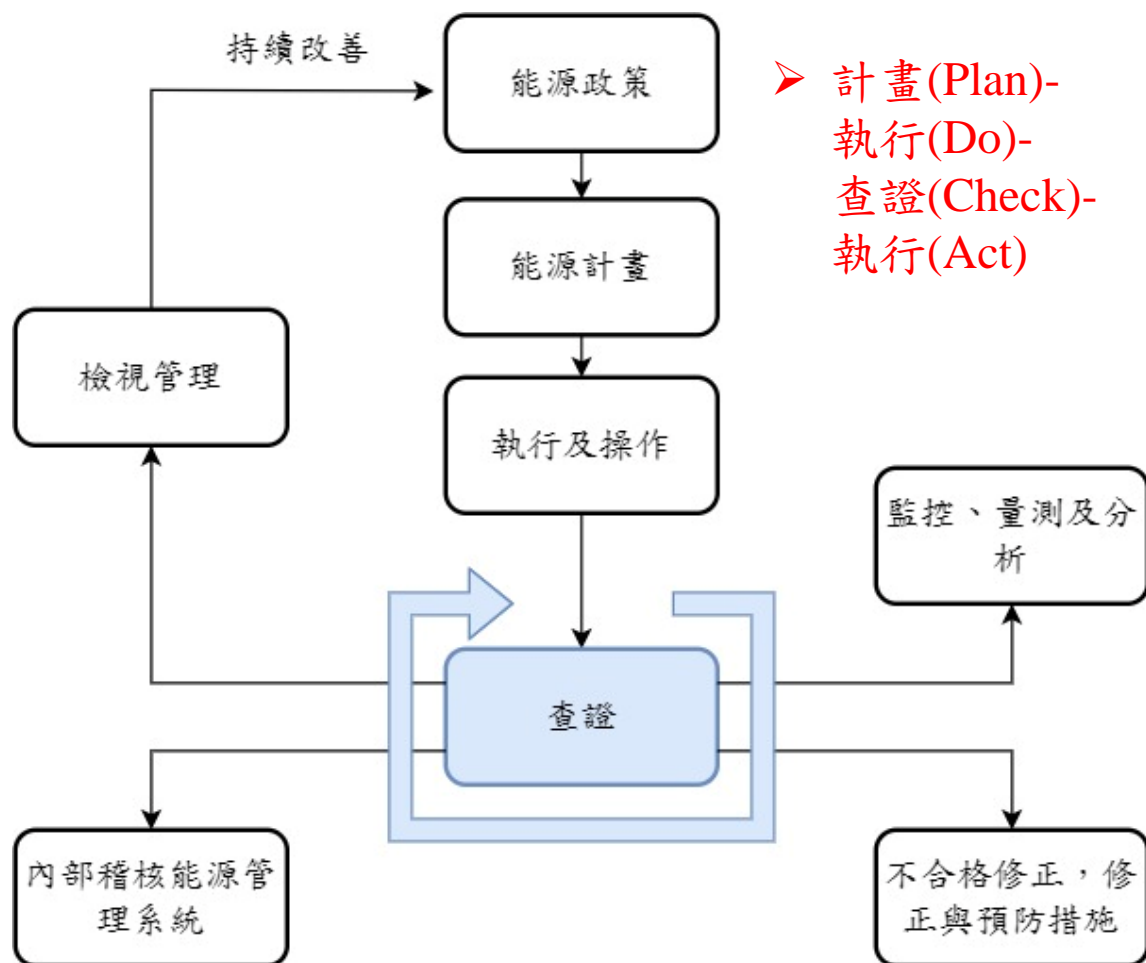
Figure 2. Savings or Avoided Energy Consumption or Demand

Ref : Core Concepts International Performance Measurement and Verification Protocol, EVO



# 量測驗證基本概念

- 能源管理系統-ISO 50001：

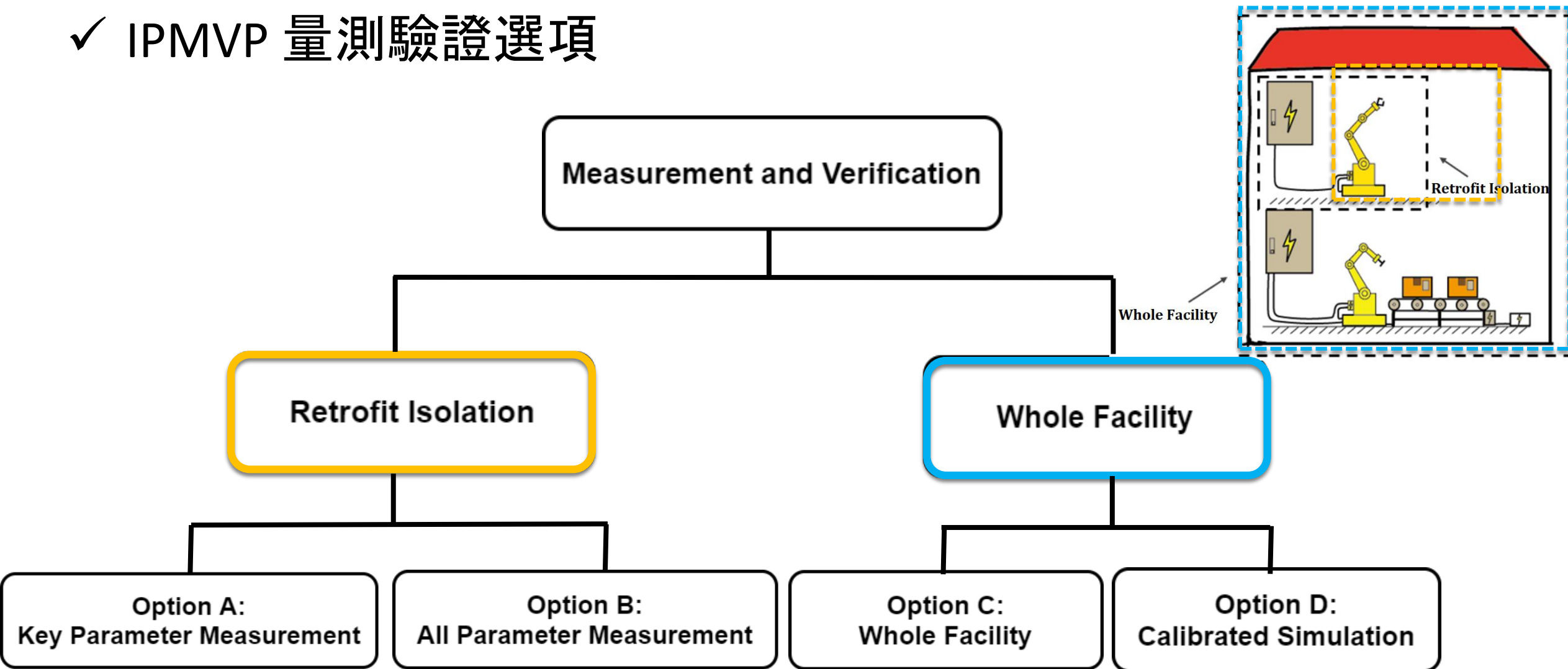


- IPMVP不包含以下細節：

- 規劃能源效率措施
- 設計量測的儀表系統
- 量測驗證成本的估算
- 能源改善工程
- 統計分析

# 量測驗證基本概念

## ✓ IPMVP 量測驗證選項



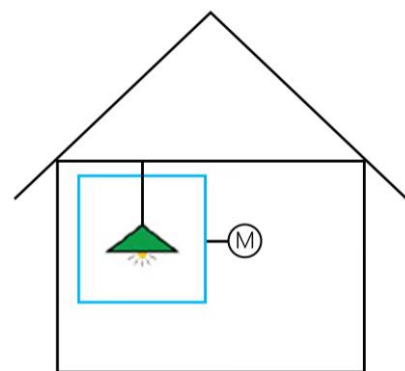


# 量測驗證基本概念

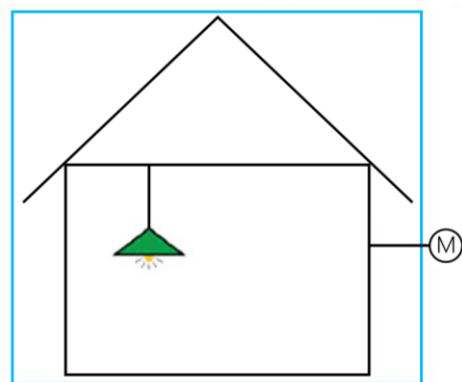
## • IPMVP四種量測選項

目前使用

改善獨立法  
量測邊界



整體設施法  
量測邊界



選項A  
改善獨立法:  
關鍵參數量測

- 量測部分與能耗相關的參數，這些參數代表節能措施成功與否
- 未選擇的參數令為估計值，基於歷史數據、製造商規格、工程判斷
- 量測頻率範圍從短期到連續，取決於測量參數的預期變化與報告週期的長度

選項B  
改善獨立法:  
所有參數量測

- 使用短期、定期或連續量測所有與能耗相關的參數
- 節能驗證結果會比選項A來的準確，但成本也較高

選項C  
整體設備法

- 通過測量整體設施的能耗和需求來確定節能
- 在整個報告期內，對整個設施內的能耗和需求進行連續測量

選項D  
校準模擬法

- 通過模擬整個設或子設施的能耗和需求來確定節能
- 模擬被證明可以充分擬設施中的實際能源使用
- 該選項需要相當成熟的校準模擬技能

# 量測驗證基本概念

IPMVP			ASHRAE Guideline 14		
Option A	Interactive effects can be reasonably estimated or ignored	<ul style="list-style-type: none"><li>- The magnitude of savings is low</li><li>- Based on a combination of measured and estimated factors</li><li>- Measurements are short-term, periodic, or continuous</li></ul>	↔	Retrofit Isolation	<ul style="list-style-type: none"><li>- Measures the energy use and relevant independent variables of the individual systems and equipment affected by the retrofit</li><li>- Determine savings of individual ECMs</li><li>- Measurement Boundary : Equipment or systems affected by retrofit</li></ul>
Option B		<ul style="list-style-type: none"><li>- Use short-term, periodic or continuous measurement of all parameters needed to calculate energy use</li><li>- Results are typically more precise using Option B than the estimations defined for Option A</li></ul>			
Option C	Interactive effects can be estimated	<ul style="list-style-type: none"><li>- Savings are predicted to be greater than about 10% to 20% of the overall consumption</li><li>- Measured by the utility or submeter on a monthly basis.</li></ul>	↔	Whole Building	<ul style="list-style-type: none"><li>- Most appropriate where the expected savings are greater than 10% of the measured energy use or demand</li><li>- Data are continuous and complete</li></ul>
				Prescriptive Building or facility	
Option D	- Whole facility or system analysis procedures to verify the performance of retrofit projects using calibrated computer simulation models - The energy model will be calibrated with past utility data		↔	Performance	<ul style="list-style-type: none"><li>- Most appropriate where the data are not continuous, have gaps</li></ul>
				Whole Building Calibrated	<ul style="list-style-type: none"><li>- Particularly suited to accounting for multiple energy end uses, especially where interactions occur between measures</li><li>- Consider use if utility data is unavailable or measurement alone cannot easily determine savings</li></ul>

Yun Mi Park, Moo Kyoung Seo, Jin Chul Park, “Analysis of Baseline Standard for Building Energy Measurement & Verification”, Korean J. Air-Cond. Refrig. Eng. Vol. 33, No. 7, p.313-319



## What is IPMVP?

- “The process of using measurements to reliably determine actual savings to reliably determine actual savings created within an individual facility by an energy management program .”

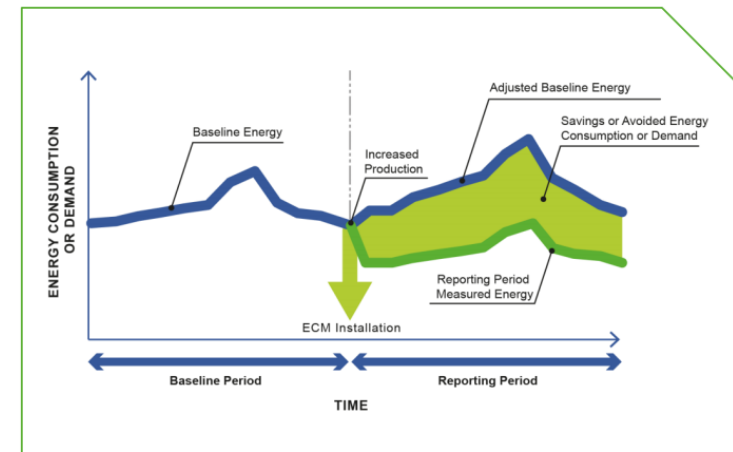


Figure 1. Savings or Avoided Consumption or Demand



## IPMVP Options

- The “best” option depends on
  - Budget
  - Metering
  - Scope
- Multiple Options may be used within an M&V Plan, depending on the considerations above.

## IPMVP Option A

- “Retrofit Isolation: Key Parameter Measurement.”
  - Baseline is an engineering calculation which could involve short-term metering, continuous metering, or an engineering estimate
  - Good for measures with consistent energy consumption (i.e. lighting, constant speed fans)

## IPMVP Option B

- “Retrofit Isolation: All Parameter Measurement ”
  - Energy use is either measured directly or a calculation which is based on a measurement
  - Good for systems with known performance curves where an isolated measurement can be used to calibrate energy consumption (i.e. VSDs)

## IPMVP Option C

- “Whole Facility”
  - Energy use metered at the whole facility level or sub-facility level
  - Good for complex EEMs which multiple systems

## IPMVP Option D

- “Calibrated Simulation”
  - Energy use is simulated with modeling software (such as eQUEST or EnergyPro) and calibrated using monthly utility bills
  - Good for facilities where no baseline data is available (i.e. new construction)

# 量測驗證基本概念

## Creating an M&V Plan

- Create a list of building systems and system components with their energy consumption characteristics (power draw, firing rates, etc.)
- Determine goals for M&V (specific ECM isolation = Options A & B, whole ECM isolation = Options C & D)
- Determine which quantities need to be metered and which can be estimated (if using Option A or B)

Table 2  
Example  
Lighting

Situation	Measurement vs. Estimation Strategy		Adherent to Option A?
	Operating Hours	Power Draw	
ECM reduces operating hours	Measure	Estimate	Yes
	Estimate	Measure	No
ECM reduces power draw	Estimate	Measure	Yes
	Measure	Estimate	No
ECM reduces both power draw and operating hours:			
Baseline power uncertain, operating hours known	Estimate	Measure	Yes
	Measure	Estimate	No
Power known but operating hours uncertain	Measure	Estimate	Yes
	Estimate	Measure	No
Power and operating hours poorly known	Measure	Estimate	No – Use Option B
	Estimate	Measure	

## M&V Plan Contents

- ECM Description, Intent, and Commissioning Procedure
- Select IPMVP **Option** and **Measurement Boundary**
- **Baseline Conditions**: Include energy use and quantify factors which affect energy use
- Identify the reporting period
- Identify whether data will be normalized (if so, savings will be “normalized savings” , if not savings will be “avoided energy use” )
- Analysis Procedure: define all mathematical models to be used and any limits to their validity.

## M&V Plan Contents M&V Plan Contents

- **Energy Prices:** Identify if they will be fixed or variable for reporting purposes.
- **Metering:** Identify meter locations and spec' s, Cx, and calibration procedures for all non all non-utility meters. Establish upper limit on acceptable data loss.
- **Monitoring Responsibilities:** Identify responsible parties for data recording and reporting.
- **Expected Accuracy:** Include meter tolerances, and uncertainty associated with sampling error and any engineering calculations.



## M&V Plan Contents M&V Plan Contents

- **Budget:** Include setup cost and on-going reporting costs.
- Report Format: Include a sample report
- Quality Assurance: Specify steps taken towards QA of savings calculation & reporting

## M&V Reporting

- **Must follow procedures identified in the M&V Plan**
- **Include:**
  - Complete data sets (justification for any “corrected” data)
  - Description and justification of adjustments
  - Any estimated values
  - Energy price schedule
  - Savings in energy and monetary units